ducting fluid implies the solutions of a coupled electromagnetic field-velocity field problem. Moreover, for very dense plasmas, it is implied that viscosity effects must be included because of their effect on the velocity field near the boundaries of the flow. Nonetheless, it is believed that the magnitudes of each are not significantly different in certain cases. For the example discussed herein, the relatively good agreement between the experimental data and the theory suggests that this is the case.

The results of the continuum theory as applied to the experimental situation do not differ in a significant way from the results of the particle approach. An essentially parabolic curve is obtained. This is primarily because of the relative predominance of the initial velocity. This velocity is an experimentally measured value. The experimental arrangement suggests, however, that this measured velocity is more representative of a portion of the ionized fraction of the plasma than of the plasma as a whole. If the velocity of the neutrals were properly weighted (this is assumed in the continuum theory), the initial velocity of the plasma would likely be lower.

At higher densities, the continuum equations represent the motion more realistically. The second term in Eq. (8) which takes into account the effects of collisions through such variables as temperature and pressure will exercise greater importance as the density increases, and hence the departure from the parabolic dependence of exit velocity on field maximum would become more pronounced. Lower initial velocities would also cause a departure from the parabolic curve.

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Three-Dimensional Supersonic Flow Computations

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THE numerical solution of systems of partial differential equations with more than two independent variables was made possible in recent years by large-scale, high-speed digital computers. However, no complete numerical analysis of three-dimensional supersonic flow fields appeared in the literature until 1961, when some of the results summarized in this note were obtained at General Applied Science Laboratories Inc.

Three-dimensional computations are difficult to perform. In building up mathematical arguments, the number of dimensions is irrelevant, but practical computations need the help of geometry, which is limited by the two-dimensionality of the paper and the blackboard. An extensive use of two-dimensional geometry allowed the author and his collaborators to achieve large-scale results in a surprisingly short time without spoiling the three-dimensional nature of the problems.

Let u, v, and w be the components of the velocity $\bar{V}$ with respect to three orthogonal axes x_1, x_2, and x_3, in order. Move to the right-hand side of the equations of motion all terms that do not contain derivatives and the terms that contain derivatives with respect to x_2. Hence, the left-hand sides of the continuity equation and of the first and third scaler momentum equations contain only ρ, p, u, and w and their derivatives with respect to x_1 and x_3. If the right-hand sides are considered as known, this system of three equations depends on x_1 and x_3 only, and its characteristics can be determined by a standard procedure. They are formally the same as in a two-dimensional problem. The compatibility equations along the characteristics can be written in a similar way, but with additional terms that involve v and the derivatives with respect to x_2. This idea, stemming on early theories,[1] is outlined in Ref. 2.

Any of the three compatibility equations and the second momentum equation can be written in the form

$$U_k + BV_k = F$$

where U and V in turn stand for $\tau(= u/w)$, $q^2(= u^2 + w^2)$, v, and p. The subscript k means differentiation along a characteristic ($k = 1,2,3$); B is a function of u, w, p, and ρ; and F is a function of these variables and of their derivatives with respect to x_2. These equations have been derived from the original equations of motion without linearization or simplifications of any kind. They suggest a finite difference technique that can be described as follows:

1) Consider one $x_3 = x_{30}$ surface and several $x_2 = x_{2i}$ lines on it. Let A_{ik} be a point on one of these lines.

2) Compute the derivatives with respect to x_2 and the other parameters in B and F at each point A_{ik}.

3) Step off the $x_3 = x_{30}$ surface along characteristics in the (x_1, x_3) surfaces, assuming B and F equal to their initial values through the first intersection of two characteristics.

The difference between the present problem and two-dimensional or axisymmetrical problems, where the assumption in step 3 is a matter of routine, consists in that here not only the parameters but also their cross derivatives must be almost constant throughout the step. A proper choice of the frame of reference is necessary. A sizeable change in some of the cross-derivatives can only occur as a consequence of an abrupt crosswise spreading of the streamlines. If the frame of reference is chosen with its curvilinear axis normal to the general direction of the flow, at each step the v component of the velocity will be small and the cross-derivatives nearly constant.

Now, the general behavior of the flow in a shock layer is a consequence of the body geometry. In the vicinity of a spherical nose, the flow is about axisymmetrical. Along a flat surface it tends to become two-dimensional. Along a rounded leading edge it tends to become cylindrical, and so forth. Consequently, different regions must be analyzed within different frames of reference.

In more sophisticated analyses dealing with three-dimensional characteristic surfaces, this problem does not arise. However, the program becomes substantially simpler if a simple, two-dimensional scheme is used throughout a region. This means that different simple frames of reference must be chosen over different regions of the body in connection with its geometry. The choice can easily be made before starting a computation. All the results have been obtained so far using a Cartesian frame and a cylindrical frame whose first meridional plane is the last Cartesian plane. The spacing between Cartesian planes, the angle between meridional planes, and the origin of the cylindrical frame can be auto-

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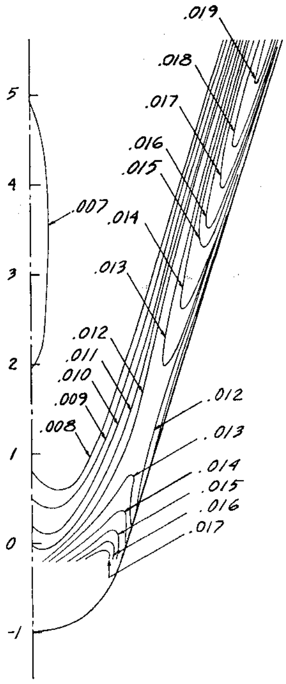

Fig. 1 Computed isobars on windward surface of blunted delta wing at $\alpha = 10°$, $M = 20$ (pressures in atmospheres).

matically changed in the course of the computation according to a scheme prescribed in advance, to follow the geometry of the body.

Full details on the program, which includes techniques for the computation of the shock wave, for the boundary condition on the body, for the evaluation of the cross-derivatives, and for real gas effects, can be found in Ref. 3. Here some significant results will be presented. The following cases have been computed:

1) Delta wing of 70° sweepback, with spherical nose and rounded leading edge, at no angle of attack, $M = 8$ (perfect gas) and $M = 20$ (real gas in equilibrium), length 10 nose radii (January 1961).

2) Blunted cone, 20° semi-apex angle at $\alpha = 5°$, $M = 8$, calculation performed all around the body, length 11 nose radii (June 1961).

3) Dynasoar body (combination of different delta wings, with different sweepbacks, different leading edges, and different inclinations) at $M = 18$, at 10° and 30° angle of attack, length 37 and 5 nose radii, respectively; computations of the windward side (September 1961).

4) Combination of sphere, cone, and cylinder; $M = 22$, $\alpha = 10°$ and 30°, length 7 nose radii, computation of the windward side for the higher angle of attack and of the whole flow field for the lower angle (April 1962).

These computations, besides proving the reliability and effectiveness of the technique, have provided interesting information from the aerodynamical and computational standpoint. Case 2 has given numerical confirmation to the experimental results that an overexpansion takes place behind the nose of the cone, followed by a recompression, and that a practically conical flow is reached at about 6 nose radii from the nose of a blunted cone. Cases 3 and 4 showed the difficulties in stepping away from the initial axisymmetric region in the case of a very high angle of attack. In the same cases the independence of the windward side from the leeward side has been successfully applied to make possible the computation only on the side of interest. In case 4 an additional technique for computing streamlines was applied.[4]

The required IBM 7090 time per step in each meridional plane is about $\frac{1}{200}$ min, using seven points on each radial line. Examples of results are given in Figs. 1 and 2.

Figure 1 shows a sample of computed isobars on the surface of the delta wing at angle of attack. Figure 2 shows the computed shape of the shock surface on the windward side of a delta wing at angle of attack.

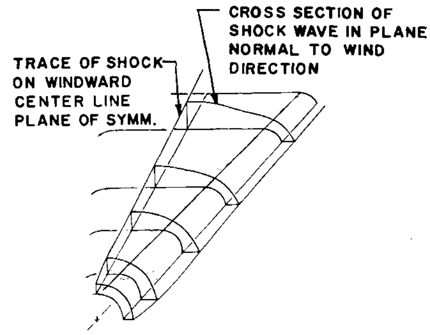

Fig. 2 Computed shock wave envelope about blunted 70° sweepback delta wing at $\alpha = 20°$, $M = 8$.

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Measurements of Heat Transfer Rates in Separated Regions in a Shock Tube and in a Shock Tunnel

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SHOCK tubes and shock tunnels have been used extensively in the last few years for heat transfer studies using their supersonic and hypersonic high enthalpy flows. It has been shown that quasi-steady heat transfer rates are established over a regular model in the shock tube within 10 to 50 μsec (e.g., results in Ref. 1). The flow starting process in the shock tunnel is more intricate. For the tailored interface shock tunnel this starting time has been measured to be about 1 msec out of the 5 or 6 msec available duration of steady flow.[2] The establishment of steady flow conditions in these facilities has been ascertained by schlieren photography and by pressure and heat transfer measurements.[1, 2] The use of these very short duration intermittent operating facilities for studies of separated flows is underway in various laboratories.[3–5] In the case of separated flows one must prove that in addition to the steadiness of the external flow, the intermittent flow duration is sufficient to establish steady mass and heat transfer conditions within the separated region. This problem has been acknowledged by those concerned in shock tube work, and the establishment of steady heat transfer in each case is discussed in Refs. 3–5. How-

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